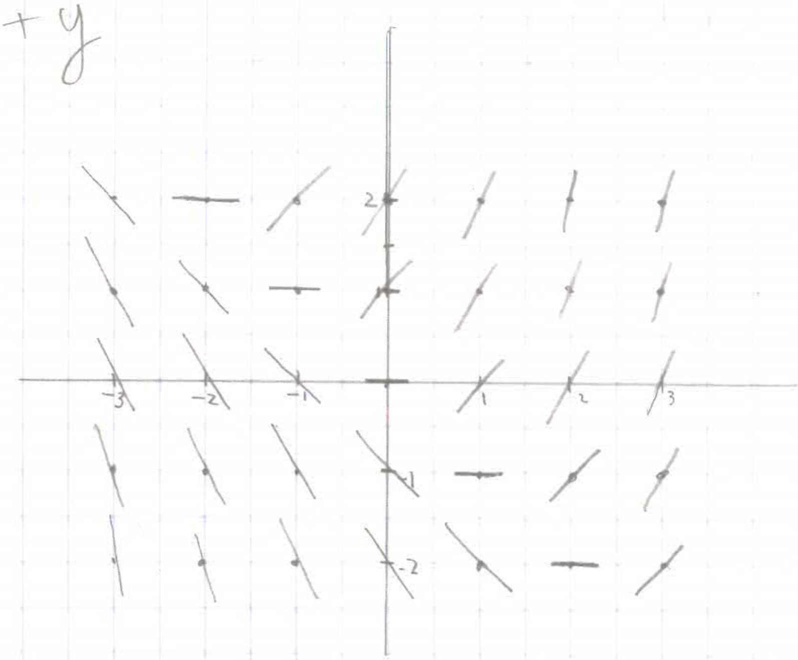
2) $\frac{dy}{dx} = x + y$

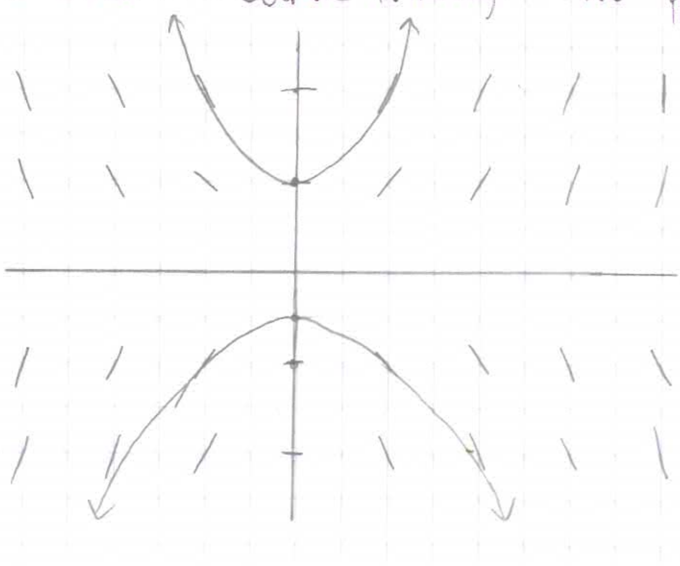


B. Solution Curves: a family of curves that is represented by the general solution of a first order differential equation. In other words, solution curves represent the actual graph predicted by the results of the dashes of the slope fields.

Example:

The calculator drawn slope field for the differential equation $\frac{dy}{dx} = xy$ is shown in the figure below. The solution curve passing through the point (0,1) is also shown.

a) Sketch the solution curve through the point (0,2)
b) Sketch the solution curve through the point (0,-1)



Sample Problems.

✓1) Analyze and sketch the graph of $f(x) = 2x^{5/3} - 5x^{4/3}$.

✓2) Analyze and sketch the graph of

$$f(x) = \frac{\cos x}{1 + \sin x}$$

✓3) Determine the open intervals on which the graph of $f(x) = x^3 - 6x^2 + 12x$

✓4) Find the length and width of a rectangle with a perimeter of 100 meters and maximum area.

✓5) Two posts, one twelve feet high and the other 28 feet high stand 30 feet high. They are to be stayed by 2 wires, attached to a single stake, running from ground level to the top of each post where should the stake be placed to use the least amount of wire?

6) Find the extrema of $f(x) = 2x - 3x^{2/3}$ on the interval $[-1, 3]$.

7) Find the extrema of $f(x) = 2\sin x - \cos 2x$ on the interval $[0, 2\pi]$.

8) Find the rate of change of the distance between the origin and a moving point on the graph of $y = x^2 + 1$ if $\frac{dx}{dt} = \frac{2\text{ cm}}{\text{sec}}$

(2.6 notes)

9) A spherical balloon is expanding. If the radius is increasing at the rate of 2 in. per min, at what rate is the volume increasing when the radius is 5 inches?

10) A balloon leaves the ground 200 m away from an observer and rises vertically at the rate of 50 meters per min. At what rate is the distance between the balloon and the observer increasing at the instant when the balloon is exactly 200 m above the ground? At that same instant, at what rate is the angle of inclination of the observer's line of sight increasing?

✓11) Find $\frac{dy}{dx}$ if $5y^2 + \sin y = x^2$

✓12) Compute $\frac{dy}{dx}$ if $x \sin y + y \cos x = \sqrt{2}$ · $\pi/4$

✓13) Given $x^2 + y^2 = 25$, find $\frac{d^2y}{dx^2}$

✓14) Find the speed and acceleration of a particle whose motion is defined by $x = 3t$ and $y = 9t - 3t^2$ when $t = 2$.

✓15) At time $t = 0$, a diver jumps from a platform diving board that is 32 feet above the water. The position of the diver is given by $s(t) = -16t^2 + 16t + 32$. (s = feet, t = seconds)

a) when does the diver hit the water? p114

b) what is the diver's velocity at the impact?

✓16) $\frac{dy}{dx} = 2x + y$ draw the slope field for this differential equation.

✓17) draw a particular solution for $\frac{dy}{dx} = 2x + y$ passing through $(1,1)$.

Answer Key.

1) Sketching the graph of a radical function (pg 212)

$f(x) = 2x^{5/3} - 5x^{4/3}$ D= all real numbers

$f'(x) = \frac{10}{3}x^{1/3}(x^{1/3} - 2)$

$f''(x) = \frac{20(x^{1/3} - 1)}{9x^{2/3}}$

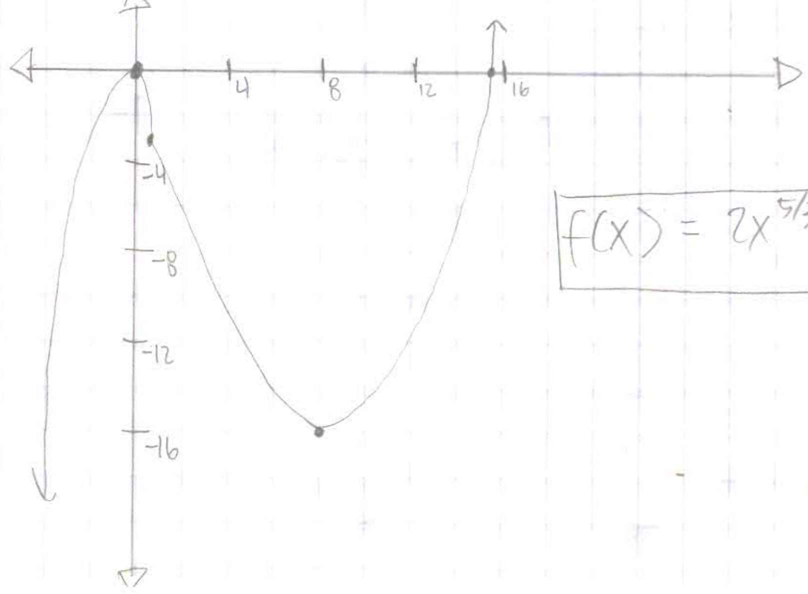
a) Intercepts = $(0,0)$ & $(\frac{125}{8}, 0)$

b) no asymptotes

c) crit. numbers = $(x=0)$ & $(x=8)$

d) P.I.P. = $(x=0)$ & $(x=1)$

	$f(x)$	$f'(x)$	$f''(x)$	Characteristic of Graph
$-\infty < x < 0$		+	−	Increasing, CD
$x=0$	0	0	und.	rel. max
$0<x<1$		−	−	decreasing, CD
$x=1$	−3	−	0	Point of Inflection
$1<x<8$		−	+	decreasing, CU
$x=8$	−16	0	+	Rel. min
$8<x<\infty$		+	+	Increasing, CU



$\boxed{f(x) = 2x^{5/3} - 5x^{4/3}}$

2) Sketching the graph of a Trig. Function

a) $f(x) = \dfrac{\cos x}{1+\sin x}$

$f'(x) = -\dfrac{1}{1+\sin x}$

$f''(x) = \dfrac{\cos x}{(1+\sin x)^2}$

b) period $= 2\pi$

c) x-int $= (\pi/2, 0)$
y-int $= (0, 1)$

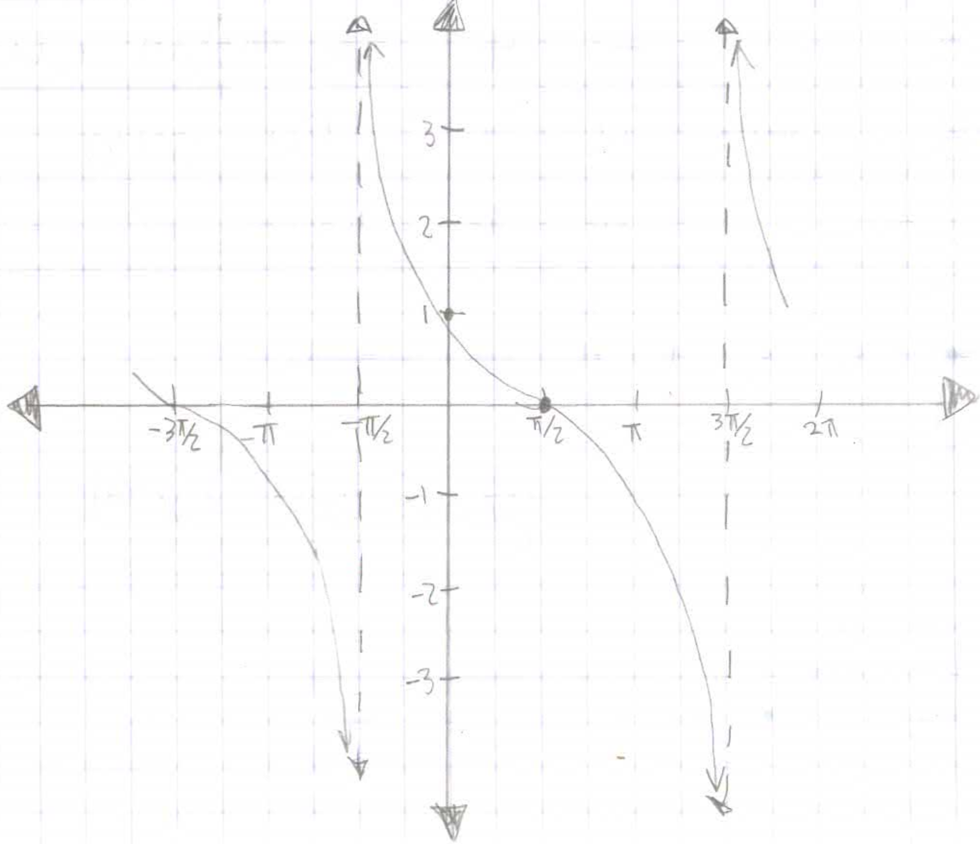
d) VA $= (x = -\pi/2), (x = 3\pi/2)$
HA = none

e) no critical numbers

f) PIP $(x = \pi/2)$

g) Domain = all real numbers except
$x = \dfrac{3+4n}{2}\pi$

	f(x)	f'(x)	f''(x)	Characteristic of Graph
$x = -\pi/2$	und	und	und	V.Asymptote
$-\pi/2 < x < \pi/2$		−	+	decreasing CU
$x = \pi/2$	0	$-1/2$	0	PIP
$\pi/2 < x < 3\pi/2$		−	−	Decreasing CD
$x = 3\pi/2$	und	und	und	V. asymptote



③ $f(x) = x^3 - 6x^2 + 12x$

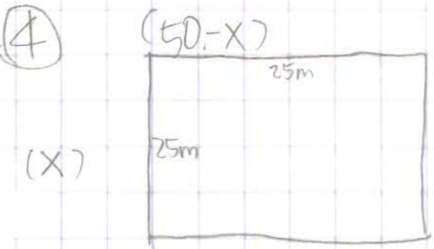
$f'(x) = 3x^2 - 12x + 12$

$f''(x) = 6x - 12 = 0$

$x = 2$

$x < 2$	2	$2 < x$
$-$	0	$+$
CD	I.P.	CU

④



$A = xy$

$A = x(50-x)$

$A = 50x - x^2$

$A' = 50 - 2x = 0$

$x = 25$

$A'' = -2 < 0 \leftarrow$ rel max

$\boxed{25m \times 25m}$

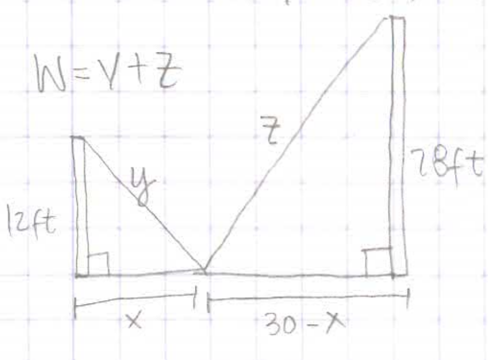
$P = 2x + 2y = 100$

$x + y = 50$

$y = 50 - x$

$W = y + z$

⑤



1) $x^2 + 12^2 = y^2$

$(30-x)^2 + 28^2 = z^2$

2) $y = \sqrt{x^2 + 144}$

$z = \sqrt{x^2 - 60x + 1684}$

3) $W = y + z$

$W = \sqrt{x^2 + 144} + \sqrt{x^2 - 60x + 1684}$ $\quad 0 \le x \le 30$

4) $\frac{dW}{dx} = \frac{x}{\sqrt{x^2+144}} + \frac{x-30}{\sqrt{x^2-60x+1684}} = 0$

5) $\left(x\sqrt{x^2-60x+1684}\right)^2 = \left((30-x)\sqrt{x^2+144}\right)^2$

6) $x^2(x^2 - 60x + 1684) = (30-x)^2(x^2+144)$

7) $x^4 - 60x^3 + 1684x^2 = x^4 - 60x^3 + 1044x^2 - 8640x + 129{,}600$

8) $640x^2 + 8640x - 129600 = 0$

$320(x-9)(2x+45) = 0$

$x = 9, \cancel{-22.5}$ not in domain

9) $W(0) \approx 53.04$

$W(9) = 50$

$W(30 \approx 60.31$

10) The wire should be staked at 9ft from the 12-foot pole.

6) $f(x) = 2x - 3x^{2/3}$

$$f'(x) = 2 - \frac{2}{x^{1/3}} = 2\left(\frac{x^{1/3} - 1}{x^{1/3}}\right)$$

$x = 1, 0$

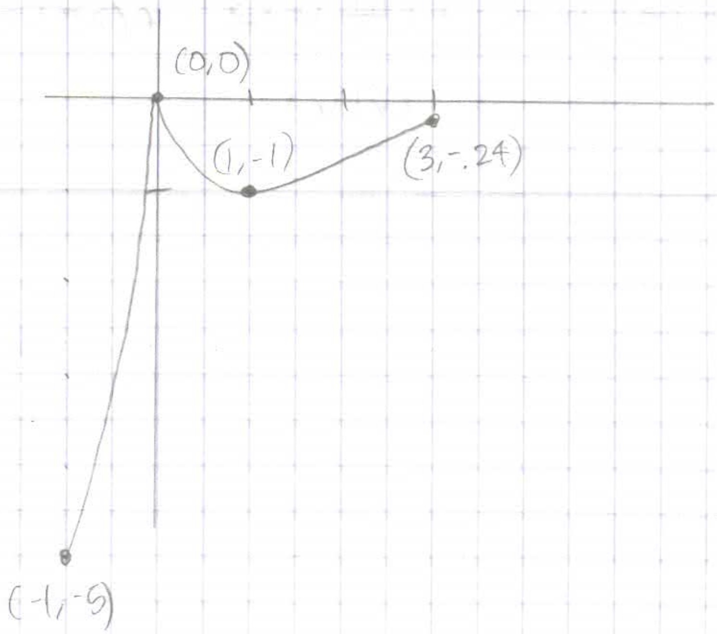
$f(1) = -1$

$f(0) = 0$ max

$f(-1) = -5$ min

$f(3) = 6 - 3\sqrt[3]{9}$

left endpt	critical #	critical #	right endpt
$f(-1) = -5$ minimum	$f(0) = 0$ maximum	$f(1) = -1$	$f(3) = 6 - 3\sqrt[3]{9}$



7) $f(x) = 2\sin x - \cos 2x$

$f'(x) = 2\cos x + 2\sin 2x = 0$

$2\cos x + 4\sin x \sin x = 0$

$2(\cos x)(1 + 2\sin x) = 0$ Factor!

$x = \underbrace{7\pi/6,\ 11\pi/6}_{\text{critical \#}}$

$f(0) = -1$

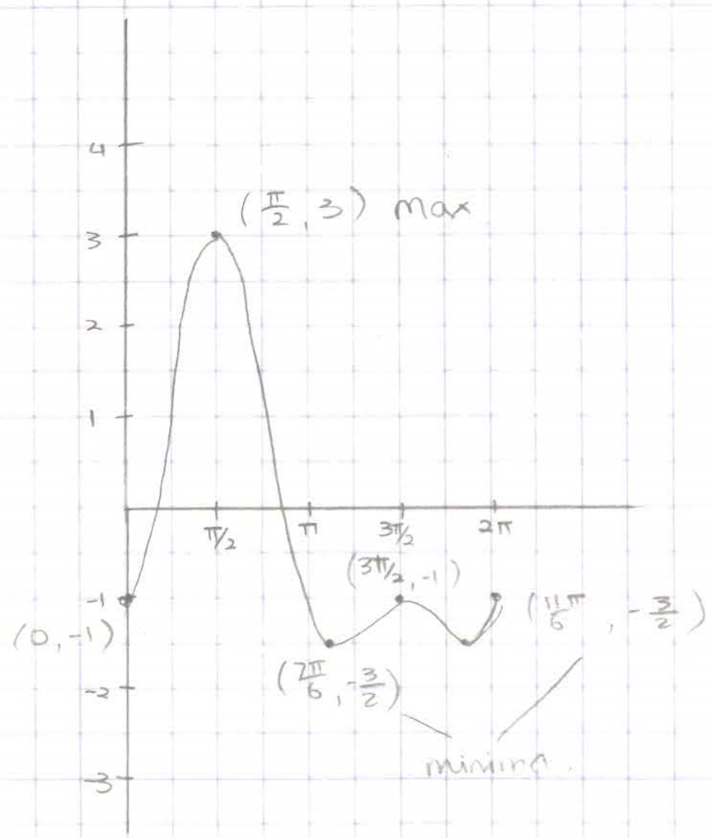
$f(\frac{\pi}{2}) = 3$ max

$f(7\pi/6) = -\frac{3}{2}$ min

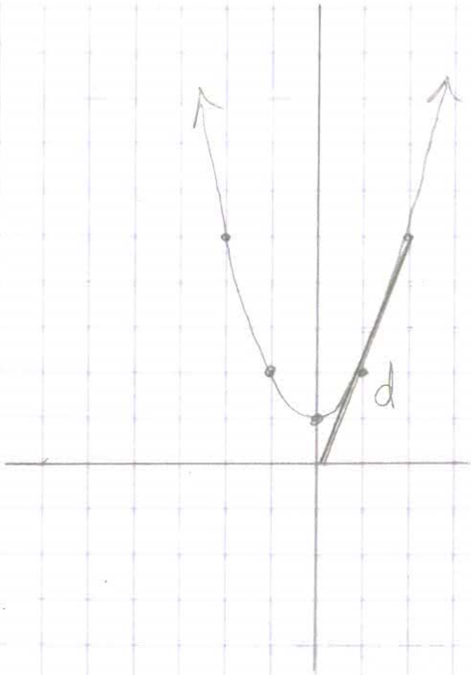
$f(11\pi/6) = -\frac{3}{2}$ min

$f(2\pi) = -1$

Left endpoint	Critical Number	Critical Number	Critical Number	Critical Number	Right Endpoint
$f(0) = -1$	$f(\frac{\pi}{2}) = 3$ Max	$f(\frac{7\pi}{0}) = 3$ min.	$f(\frac{3\pi}{2}) = -1$	$f(\frac{11\pi}{6}) = -\frac{3}{2}$ min	$f(2\pi) = 1$



8)



$y = x^2 + 1$

$\frac{dx}{dt} = 2 \frac{cm}{sec}$

$d^2 = x^2 + y^2$

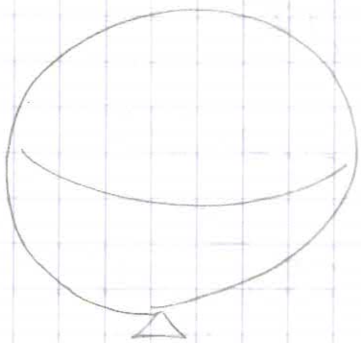
$d^2 = x^2 + (x^2+1)^2$

$2d \frac{dd}{dt} = 2x \frac{dx}{dt} + 2(x^2+1)$

$$\frac{\left(2x \frac{dx}{dt}\right)}{2}$$

$d \frac{dd}{dt} = \frac{2x + 4x(x^2+1)}{\sqrt{x^2 + (x^2+1)^2}}$

9)



$\frac{dR}{dt} = 2 \frac{in}{min}$

$\frac{dV}{dt} = ?$

$R = 5 in$

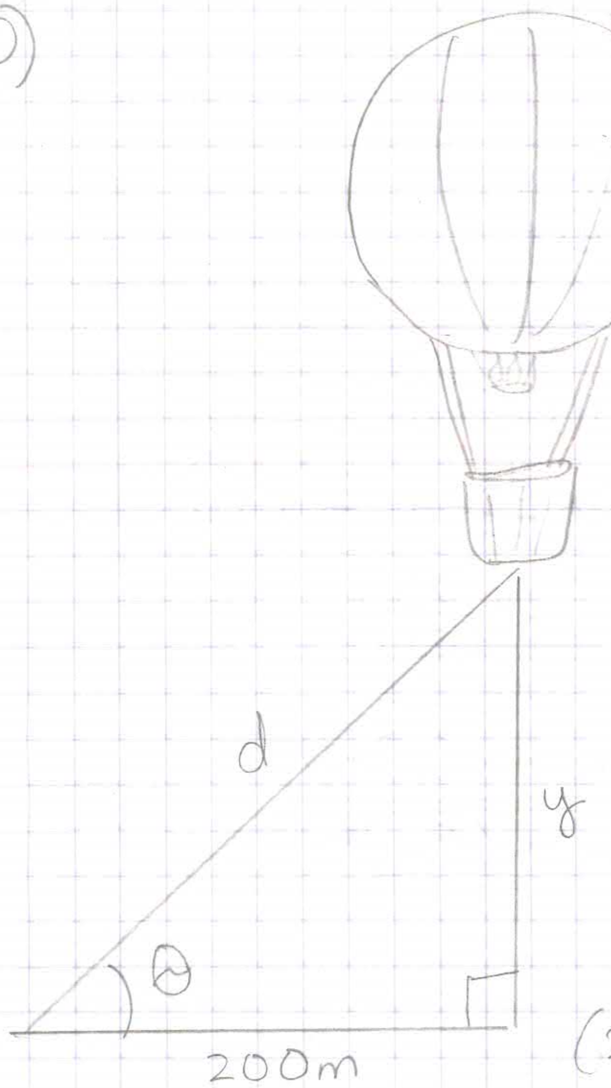
$V = \frac{4}{3} \pi R^3$

$\frac{dV}{dt} = 4\pi R^2 \frac{dR}{dt}$

$\frac{dV}{dt} = 4\pi (5)^2 \cdot 2$

$\frac{dV}{dt} = 200\pi \frac{in^3}{min}$

10)



$$\frac{dy}{dt} = 50 \frac{\text{meters}}{\text{min}}$$

$$d^2 = y^2 + (200)^2$$

$$2d \frac{dd}{dt} = 2y \frac{dy}{dt}$$

$$(200\sqrt{2}) \frac{dd}{dt} = 200 \cdot 50$$

$$\frac{dd}{dt} = \frac{50}{\sqrt{2}} = 25\sqrt{2} \frac{m}{\text{min}}$$

$$\tan\theta = \frac{y}{200}$$

$$\tan\theta = \frac{1}{200} y$$

$$\sec^2\theta \cdot \frac{d\theta}{dt} = \frac{1}{200} \cdot \frac{dy}{dt}$$

$$(\sqrt{2})^2 \cdot \frac{d\theta}{dt} = \frac{1}{200} \cdot 50$$

$$\frac{d\theta}{dt} = \frac{1}{8} \frac{\text{radian}}{\text{mins}}$$

(11) $5y^2 + \sin y = x^2$

$10y\frac{dy}{dx} + \cos y\frac{dy}{dx} = 2x$

$\frac{dy}{dx}(10y + \cos y) = 2x$

$$\boxed{\frac{dy}{dx} = \frac{2x}{10y + \cos y}}$$

(12) $x\sin y + y\cos x = \sqrt{2} \cdot \pi/4$

$1\cdot(\sin y) + x(\cos y\frac{dy}{dx}) + \frac{dy}{dx}(\cos x) + y(-\sin x) = 0$

$\frac{dy}{dx}(x\cos y + \cos x) = -\sin y + y\sin x$

$$\boxed{\frac{dy}{dx} = \frac{-\sin y + y\sin x}{x\cos y + \cos y}}$$

(13) Finding the 2nd derivative Implicitly

1) $x^2 + y^2 = 25$ find 2nd derivative

$2x + 2y\frac{dy}{dx} = 0$

$2y\frac{dy}{dx} = -2x$

$\frac{dy}{dx} = \frac{-2x}{2y} = \frac{-x}{y}$

2) $\frac{d^2y}{dx^2} = -\frac{(y)(1) - (x)(dy/dx)}{y^2}$

$= -\frac{y - x(-x/y)}{y^2}$

$= -\frac{y^2 + x^2}{y^3}$ → 25

$= \boxed{-\frac{25}{y^3}} = \frac{d^2y}{dx^2}$

(14) $\frac{dx}{dt} = 3 \quad \frac{dy}{dt} = 9 - 6t$

$t = 2 \quad \frac{dx}{dt} = 3 \quad \frac{dy}{dt} = -3$

$\sqrt{3^2 + (-3)^2} = \sqrt{18} = 3\sqrt{2} \ \frac{m}{s}$

$\frac{d^2x}{dt^2} = 0 \quad \frac{d^2y}{dt^2} = -6$ acceleration vector is $\{0, -6\}$

15) $S(t) = -16t^2 + 16t + 32$ = position func

a) Time diver hits water

$-16t^2 + 16t + 32 = 0$

$-16(t+1)(t-2) = 0$

$t = -1$ or 2 $\qquad t \geq 0$

$\boxed{\text{Diver hits water at } t = 2 \text{seconds}}$

b) Diver's velocity @ impact

$S(t) = -16t^2 + 16t + 32$

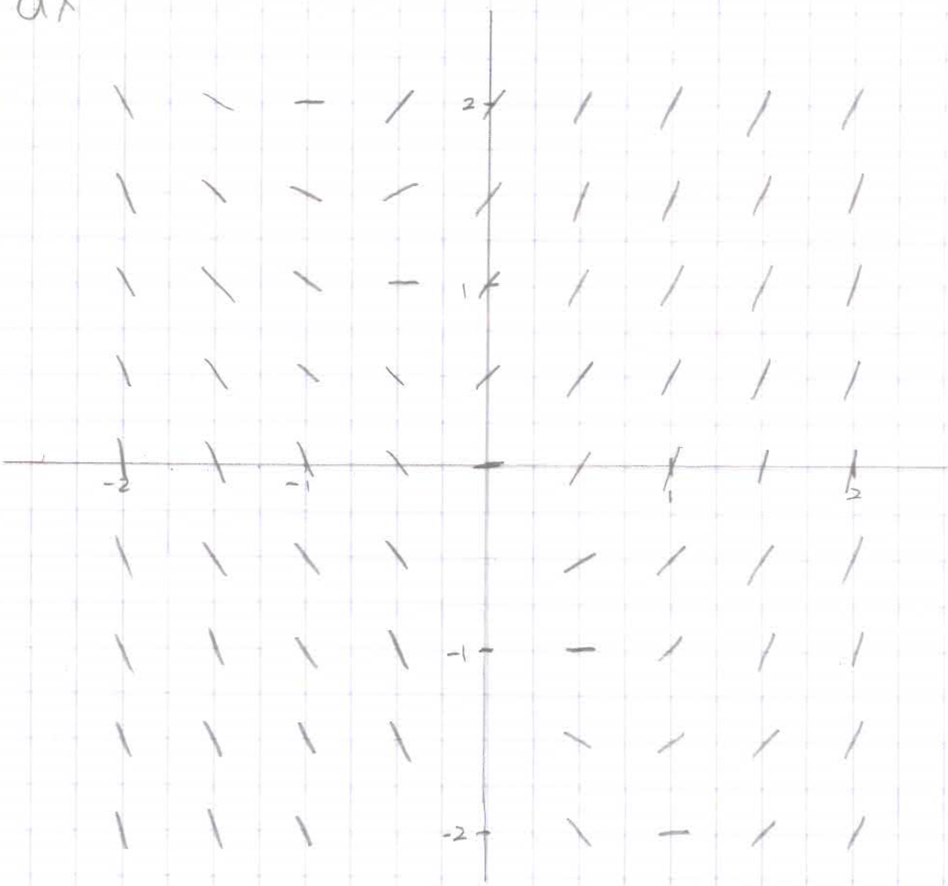
Velocity → $S'(t) = -32t + 16$

$S'(2) = -32(2) + 16$

$\boxed{-48 \text{ ft/sec.} = \text{diver's velocity at impact}}$

per

16) $\frac{dy}{dx} = 2x + y$



17)